

TRANSLATION AND THE GENESIS OF MATHEMATICS IN GREECE

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1. Greek mathematics: Autochthonous or inspired from abroad?

Herodotos (*Histories* II, 109 [1920, I, 396–398]) and numerous other Greek authors tell that geometry was first created by Egyptian surveyors (“rope stretchers”); Proclus [1873, 65], probably borrowing from Eudemos, adds that “precise knowledge of numbers” (τῶν ἀριθμῶν ἀκριβῆς γνῶσις) was developed by the Phoenicians. General Greek lore then ascribes to Thales, Pythagoras and other early Greek mathematicians the import and transformation of this foreign material into a theoretical structure.

How much of this can be relied upon – and how it is to be understood – is difficult to know; there is no doubt that the Greeks did learn from their neighbours, and that even the “Pythagorean theorem” was known (if not exactly as a “theorem”) in Babylonia around 1800 B.C. But apart from a fragment from the hand of Hippocrates of Chios which *may* be genuine but is more likely to be Eudemos’s paraphrase [cf. Knorr 1986, 38f.; Høyrup 1990b, 214], only a few hints in Plato and scattered phrases in the *Corpus aristotelicum* give us the words of Greek mathematicians prior to Autolykos (c. 300 B.C.). We may still hold that a conceptual import must have involved “translation” at least in some general sense, but the source material is clearly insufficient to make such a claim informative.

A step toward informed and informative discussion was made by Neugebauer [1936, 250], who spoke explicitly of “translation”:

Die Antwort auf [...] die Frage nach der geschichtlichen Ursache der Grundaufgabe der gesamten geometrischen Algebra [i.e., the application of an area with deficiency or excess, *Elements* II.5–6, the core of the book], kann man heute vollständig geben: sie liegt einerseits in der aus der Entwicklung der irrationalen

Größen folgenden Forderung der Griechen, der Mathematik ihre Allgemeingültigkeit zu sichern durch Übergang vom Bereich der rationalen Zahlen zum Bereich der allgemeinen Größenverhältnisse, andererseits in der daraus resultierenden Notwendigkeit, *auch die Ergebnisse der vorgriechischen "algebraischen" Algebra zu übersetzen.*

Hat man das Problem in dieser Weise formuliert, so ist alles Weitere vollständig trivial und liefert *den glatten Anschluß der babylonischen Algebra an die Formulierungen bei Euklid.*

Two ideas go into these formulations: That the Greek discovery of incommensurability would have provoked a "foundation crisis" in Greek geometry – an idea set forth by Hasse and Scholz [1928] – by disproving a Pythagorean identification of geometric entities and integer numbers; and Neugebauer's and his collaborators' discovery that a number of Babylonian clay tablets (most of them from the second half of the Old Babylonian period, 1800–1600 B.C., some from the Seleucid era, 3rd and 2nd c. B.C.) contain problems which correspond closely to modern mixed second-degree-equations (i.e., equations of the types $\alpha x^2 + \beta x = \gamma$, α , β , and γ positive or negative), but which are often formulated in terms closer to *Elements* II.5–6, *viz.* as dealing with rectangles for which the area and the sum of the sides or their difference are given.

For a while, Neugebauer's thesis was generally accepted; serious discussion was only started by Arpád Szabó and Sabetai Unguru. Szabó [1969, 455ff.] formulated two objections: (1) that the early Greek geometers are not likely to have known about the contents of clay tablets they could not read; (2) that not only the aims of Greek and Babylonian mathematics were totally different (the finding of theorems, and the construction of figures, versus the finding of numerical solutions) but also the conceptualizations of the subject-matter – whereas Babylonian "algebra" as understood by Neugebauer was a purely numerical discipline in spite of its use of a geometrical terminology, *Elements* II are geometrical through and through. He did not find that the postulated *Grundlagenkrise* was able to bridge this difference, and thought it much more likely that the area geometry of *Elements* II was an autochthonous development, whose starting point was similar to the heuristic geometry told about in Plato's *Meno* (82B–85E) – the passage where a slave boy is brought to understand how to double a square. Unguru [1975; cf. Unguru & Rowe 1981] rejected the idea

of translation from a numerical algebra even more vehemently, as based on and entailing a modernizing reading that obscured the nature and aims of the Greek mathematicians instead of elucidating them.

2. Another reading of Babylonian “algebra”

A new understanding of Babylonian “algebra” and of its connections to later traditions has allowed to reassess the Neugebauer thesis. In this interpretation [cf. Høyrup 1990a], the “sides” and “areas” of its problems are understood geometrically, not as metaphors for numbers and their products; the operations are read as cut-and-paste transformations of geometric configurations. As an example we may look at Figure 1, configuration (I), which represents two different problems: Either a rectangle with known area A , where the difference between length and width is given, $l-w = \delta$, or a square $\square(s)$, where the sum of the area and δ sides ($\square(s)+\delta \cdot s$) equals A . In both cases, A

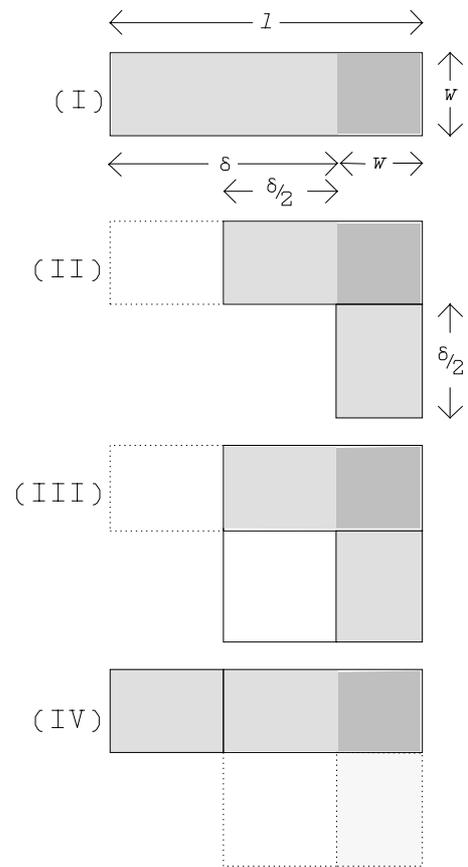


Figure 1

represents the total shaded area. The segment δ is “broken” (bisected) and the outer part moved (II) so as to “hold” together with the inner part a square $\square(\delta/2)$ of known area; this quadratic complement (white) is joined to the shaded gnomon (III), which becomes a square with known area

$A+(\delta/2)^2$, and hence also known side $S = \sqrt{A+(\frac{\delta}{2})^2}$. The width w of the

rectangle (or the side s of the square) is found by removing the semi-rectangle which was moved downwards in (II), $w = S-\delta/2$, and the length l of the rectangle by putting it back in its original position, $l = S+\delta/2$. No formal proof is given, but the procedure is immediately “seen” to be correct; we may characterize it as “naive”, in contrast to the “critical” approach of *Elements* II, to which we shall return. The style of the argument

is precisely that of the *Meno* passage. The numerical steps coincide with those of modern equation algebra – the fact that once caused the modernizing reading of Neugebauer and others; phrases and conceptual distinctions made in the texts which did not fit this reading were disregarded or explained away.

Problems where we know the difference between a square area and δ sides are solved by means of the same geometric operations, rectangle problems where the area and the sum of the sides is known by means of a different but analogous configuration.

3. A practical-geometrical tradition

This algebraic technique was developed to great sophistication in the Old Babylonian scribe school, but it was not invented there. Its origin seems to be in an environment of non-scholastic practical geometers (surveyors, master-builders, etc.). In this – probably Akkadian-speaking – environment, a number of geometrical riddles seem to have circulated in the late third millennium B.C. After the collapse of the Neosumerian state around 2000 B.C., when Akkadian became the language of the scribe school, these riddles were adopted into the curriculum, where they served as the foundation of a sophisticated mathematical discipline. Around 1600 B.C., however, the Old Babylonian scribe school disappeared, as did its sophisticated mathematics; the “algebra” which turns up in late Babylonian tablets is closer to the original riddle type than to the refined school discipline. Nothing positive is known about the transmission channels, but both the oral teaching of geometrical practitioners’ apprentices and scribe schools of the Mesopotamian periphery are likely to have played a role.

In the ninth century, the riddles and the cut-and-paste technique turn up in various Arabic sources: the latter serves in al-Khwārizmī’s geometric proofs for his solutions of second-degree equations; riddles and solution technique form the bulk of the first chapters of Abū Bakr’s *Liber mensurationum* (known only in Gerard of Cremona’s 12th-c. Latin translation).

Much of this story has to be reconstructed from indirect evidence – see [Høyrup 1996]; in particular, combination of the evidence from the various epochs permits a reconstruction of the basic stock of riddles. If $4s$ designates “all four sides” of a square and d the diagonal of a rectangle

(other symbols as above), a single square gave rise at least to these problems (α and β stand for given numbers):

$$s+A = \alpha, \quad 4s+A = \alpha; \quad A-s = \alpha;$$

$$\text{perhaps } 4s-A = \alpha; \quad \text{probably } 4s = A;$$

the standard solution appears to have been $s = 10$ when mathematically possible. When linguistically possible, the side was mentioned before the area; since the area is a derived entity, this order agrees with general riddle style.

Two squares give rise to four problems:

$$A_1+A_2 = \alpha, \quad s_1\pm s_2 = \beta; \quad A_1-A_2 = \alpha, \quad s_1\pm s_2 = \beta.$$

For rectangles, these problems occur:

$$A = \alpha, \quad l\pm w = \beta; \quad A+(l\mp w) = \alpha, \quad l\pm w = \beta; \quad A = \alpha, \quad d = \beta.$$

The circle (diameter d , perimeter p , area A) gives occasion at least to this problem:

$$d+p+A = \alpha.$$

“Unnatural” coefficients (“twice the area”, “a third of a side”, etc.) appear to have been avoided; in scribe school texts, on the contrary, they abound.

In practical calculations, two important quasi-algebraic identities seem to have been used in the late third millennium:

$$\square(S+\sigma) = \square(S)+\square(\sigma)+2\sqcap(S,\sigma); \quad \square(S-\sigma) = \square(S)-2\sqcap(S,\sigma)+\square(\sigma).$$

These formulae facilitate the determination of the areas of squares whose sides differ by a small amount σ from a round measure S .

4. The several levels of Greek mathematics

Neugebauer’s thesis regards the “scientific” level of Greek geometry, while Babylonian algebra was the product of a scribe school, where “mathematics” was always a technique for finding numerical solutions; this accounts for some of the differences pointed out by Szabó and Unguru, but does not eliminate their objections. Before asking how the new reading of the Babylonian texts transforms the picture we should remember that even the Greek and Hellenistic world had their mathematical practitioners. Since practitioners, not clay tablets, are the likely sources for the theoretical geometers, they are important for our argument.

The mathematical practitioners of the classical world are much less visible in the sources than Euclid, Archimedes and Apollonios. They were,

indeed, culturally subliminal to such a degree that Byzantine scholars tended to ascribe all material on practical geometry to Hero, the only name they knew – and to such a degree that modern historians of mathematics persevere in speaking of *Geometrica* as Hero’s work, even though Heiberg [Hero 1914, xxi] refutes this ascription rather emphatically.

5. Greek geometrical practice

Closer analysis shows that *Geometrica* as edited by Heiberg is a modern conglomerate composed from three Byzantine conglomerates which share little more than a common background in general Near Eastern and Hellenistic geometrical practice – see [Høyrup 1997]; of particular interest is chapter 24 in Heiberg’s edition (one of the independent constituents, itself composite), problem 3 of which runs as follows [Hero 1912, 418, trans. JH]:

A square surface having the area together with the perimeter of 896 feet. To get separated [διαχωρίζω] the area and the perimeter. I do like this: In general [καθολικῶς], place outside [εκτίθημι] the 4 units, whose half becomes 2 feet. Putting this on top of itself [ποίησον ἐφ’ ἑαυτά] becomes 4. Putting together just this with the 896 becomes 900, whose squaring side [πλευρὰ τετραγωνική] becomes 30 feet. I have taken away underneath (ὑφαίρειω) the half, 2 feet are left. The remainder becomes 28 feet. So the area is 784 feet, and let the perimeter be 112 feet. [...].

This is one of the favourite riddle problems, “the four sides and the area”, found in an Old Babylonian text, in Abū Bakr, in Fibonacci’s *Practica geometrie*, and still in Pacioli’s *Summa de arithmetica*. Some of the words are identifiable as translations: διαχωρίζω corresponds to the *berûm* used in the same function in Babylonian texts, and καθολικῶς to *semper* in Gherardo’s translation of Abū Bakr. Πλευρὰ τετραγωνική corresponds to the Sumerian *í b . s i₈* used in the Babylonian texts and may be a translation too, but since the Arabic texts use an Indian loan translation (*jidr*, “root”) we have no direct evidence for the terminology of the tradition as encountered by the Greeks. The same holds for the expression ποιῆω ἐπί, “to put on top of”, also used by Hero in *Metrica* when arithmetical calculations are expressed in geometrical terms (with or without geometrical understanding).

Other features of the text may be paraphrases or explanations

introduced by the Greeks during the reception phase; all other versions of the problem speak of “the/all four sides” instead of the “perimeter” (except the ninth-century Jaina mathematician Mahāvīra’s *Ganita-sāra-saṅgraha*, which shares other puzzling features and interests with *Geometrica* 24); nowhere else do we find the suggestive “place outside” (some Babylonian texts use different explanatory devices), nor a specification that the outer half of the rectangle representing the four sides is taken away ὑπό, “underneath” (by which we are told to turn Figure 1 90° anti-clockwise). All non-Greek expressions of the tradition use the second person singular or the imperative in the prescription, as do most of the *Geometrica* constituents.

Problem 46 of the same chapter asks for the separation of a circular diameter, perimeter and area when their sum is given; even this problem is known from an Old Babylonian text, and recurs in Mahāvīra and in an Arabic treatise from c. 1200 C.E. Here, however, καθολικῶς occurs alongside of πάντοτε, “at all times”, used in the same function; separation is ἀποδιαστέλλω; still other problems use παντός/“always” to express generality. One of the other treatises brought together in *Geometrica* (mss. A+C) uses ἀεί, “always”, in the function of καθολικῶς, and διαστέλλω for separation; πάντοτε is used in two other constituents (chapter 22, and mss. S+V). 24.6 uses the purely arithmetical πολλαπλασιῶ/“I make multiple” for multiplication.

The similarities suffice to show that Greek practitioners had encountered the Near Eastern geometrical tradition and undertaken a genuine though free translation of some of its material. The divergent ways to express the same borrowed idea suggest that this translation may have been a repeated process; in any case, however, the material was paraphrased and digested before reaching the Byzantine school and copyist-scholars.

6. The circular perimeter

The same process of translation and revision is reflected in one of the several ways in which the *Geometrica*-treatises express the circular perimeter in terms of the diameter. Old Babylonian texts find the perimeter concretely, by “repeating [the diameter] until 3” (*ana 3 esēpum*) or by “tripling” (*šullušum*); they never employ the usual term for multiplication (*našûm*),

although this term is used invariably (often directly afterwards) when the area is found as $\frac{1}{12}$ times the square on the perimeter.

In *Metrica* I.xxx [Hero 1903: 74], Hero refers to those who take “the diameter to encompass the triple (τριπλάσιος) of the diameter of the circle”, and in I.xxxi (*ibid.*) to those who take it instead to be “the triple of the diameter of the circle, and a 7th part larger”. Both groups, it appears from the context, are anonymous practitioners active well before Hero’s times; Hero’s own Archimedean formulation (I.xxvi, [1903, 66]) tells that the perimeter is found as the seventh part of the diameter times (ἐπί) 22.

This is not significant in itself; if we look at the many formulae for the same matter in the *Geometrica* manuscripts, we find Hero’s own formula expressed in Hero’s way; however, we also find the “triple [τριπλάσιον or τρισόκις] and a seventh part”, but never a multiplication by 3 expressed like other multiplications. Moreover, the triple is always calculated separately, before the extra $\frac{1}{7}$ is found and added. (The same formulation is used in the spurious proposition 2 of Archimedes’s *Measurement of the circle*, but since we know nothing about the origin of this late insertion it tells us nothing more than the *Geometrica*, whose characteristic vocabulary it shares).

A similar formulation is still found in the master builder Mathes Roriczer’s *Geometria deutsch* (c. 1488): The perimeter of a circle is found by drawing the circle thrice, and dividing one of them in 7 parts, of which one is added to the three circles [Roriczer 1977, 120f.]. There seems to be no doubt that Greek or Hellenistic practitioners took over the Babylonian expression, translating very faithfully a term for concrete repetition, and added a correction in response to Archimedes’s calculation instead of changing the whole formulation. This formula was then transmitted through the European Middle Ages – probably together with a very concrete manual repetition procedure that could stabilize the wording.

7. *Elements* II – a critique

The kind of “translation” involved in the development of Greek *theoretical* geometry is wholly different. We may start by looking at *Elements* II.1–10, remembering that the contents of *Geometrica* establishes beyond

doubt that the Greeks did know about the Near Eastern tradition – the manuscripts are of Byzantine date, it is true, but the formulations exclude a post-classical import.

In symbolic translation (which does not do justice to the text, but cf. the quotation of II.6 below), the propositions tell the following ($\square(l,w)$ stands for the rectangle contained by l and w):

1. $\square(a,p+q+\dots+t) = \square(a,p) + \square(a,q) + \dots + \square(a,t)$.
2. $\square(a) = \square(a,p) + \square(a,a-p)$.
3. $\square(a,a+p) = \square(a) + \square(a,p)$.
4. $\square(a+b) = \square(a) + \square(b) + 2\square(a,b)$.
5. $\square(a,b) + \square(a/2) = \square(a+b/2)$.
6. $\square(a,a+p) + \square(p/2) = \square(a+p/2)$.
7. $\square(a+p) + \square(a) = 2\square(a+p,a) + \square(p)$.
8. $4\square(a,p) + \square(a-p) = \square(a+p)$.
9. $\square(a) + \square(b) = 2[\square(a+b/2) + \square(a-b/2)]$.
10. $\square(a) + \square(a+p) = 2[\square(p/2) + \square(a+p/2)]$.

If the rectangles and squares are understood as products and the letters as numbers instead of line segments, these become algebraic identities. This is why the technique was interpreted as “geometric algebra” by Zeuthen [1886: 5ff], in agreement with Tannery and with a tradition that can be followed back at least to Jordanus of Nemore in the early thirteenth century; and this is why Neugebauer understood the Euclidean theorems as translations of Babylonian numerical knowledge, duly provided with proofs in the Greek manner.

The reading of the Babylonian texts as geometric and reasoned though “naive” changes this relation; As an example we may look at the proof of II.6, “If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole with the added straight line and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line” [Euclid 1926, 385, trans. Heath]. The first half of the proof constructs the square on CD, where C is the mid-point of AB; draws the diagonal DE; draws BG parallel to DF and intersecting

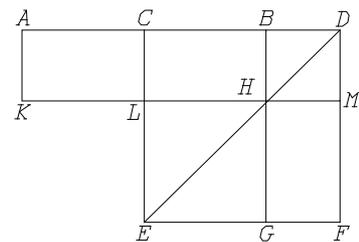


Figure 2

the diagonal in H; etc. Then it is able to prove the equality of the rectangles AL and HF, etc.

Euclid's text does not solve a problem; what it does is to make a "critique" (in quasi-Kantian sense) of the traditional naive technique, showing that what was traditionally "seen" to be correct can in fact be proved according to the best standards of theoretical geometry. But apart from the insertion of the argument in a deductive structure, where earlier propositions are made use of, the basic idea of the proof remains the same.

All the propositions have this character. II.1 shows that rectangles can be cut and pasted; II.2 and 3 treat the particular situation where sides are subtracted from or added to a square area; II.4 and 7 correspond to the two identities by which the areas of squares with "almost-round" sides were calculated since the third millennium, $\square(S+\sigma) = \square(S) + \square(\sigma) + 2\sigma S$; $\square(S) = \square(S-\sigma) + 2\sigma S - \square(\sigma)$. II.5 corresponds to the solution of the rectangle problem $A = \alpha$, $l+w = \beta$ (and of the square problem $\alpha s - A = \beta$), while II.6 corresponds to the rectangle problem $A = \alpha$, $l+w = \beta$ and to the solution of square problems $A \pm \alpha s = \beta$. II.8 shows that the difference between two square areas is four times the rectangle contained by the average side and the semi-difference between the sides, which will have served the solution of the two-square problems $A_1 - A_2 = \alpha$, $s_1 \pm s_2 = \beta$ (such problems have been found in Old Babylonian tablets, but without solution; medieval sources contain the solution); II.9 and 10 correspond to the two-square problems $A_1 + A_2 = \alpha$, $s_1 \pm s_2 = \beta$.

The proof ideas of propositions 1 through 7 correspond to what is known from Babylonian texts and is likely to have followed the practitioners' tradition since the earliest second millennium; the proof idea of II.8 may be old too, but is likely to have been revised (the practical geometers would probably locate one square concentrically within the other, which makes the idea more intuitively obvious but complicates the exact formulation). The proofs of II.9 and 10 are definitely non-Babylonian. However, the very characteristic diagram used in the proof of II.10 turns up in one of the components of *Geometrica* (mss. A+C, 16.44, [Hero 1912, 330f.]) though serving there only as a pretext for an area calculation. Even the proof ideas of II.9 and 10 are thus likely to have been borrowed from practitioners.

Propositions 4–7 are used later in the *Elements* (in particular in Book X), but the others are not: the knowledge they contain is considered so familiar that there is no reason to mention it explicitly once its reliability has been established [cf. Mueller 1981, 301]. *Elements* II.1–10 is evidently not meant to open new land but to go carefully over and thus consolidate the well-known (the same appears to hold for the rest of the book, but this requires further arguments) – to be a “critique of mensurational reason”, showing why and under which conditions (e.g., really right angles) the traditional ways could be accepted. This interpretation is corroborated by the curious fact that the proofs of the single propositions are independent, even though some of them could easily be proved by means of the preceding – 2 and 3 are special cases of 1, 6 coincides with 5 if only $b = a+p$; that each proposition gets its own proof shows that not only the knowledge contained in the theorems but also the traditional heuristic proofs were meant to be consolidated by theoretical critique.

On the whole, even Neugebauer’s thesis is hence consolidated though in revised shape. First of all, it is not the contents of the Babylonian clay tablets which is directly translated (which was never Neugebauer’s idea – in [1963, 530] he argues that the Babylonian heritage had become “common mathematical knowledge all over the ancient Near East”). Next, there is no “translation” from numbers to geometry, what we have traced is a transfer from the “naive” reason of everyday to the level of deductive and axiomatic theory. The moving force is no “foundation crisis” but the general philosophical drive of Greek thought: “not to tell us what to do, but to show why what we are doing anyway is in accord with proper principles”, as Joan Robinson formulated the task of the economist [1964, 25]. This transfer involves not only a change of proof style and of formulations – one may compare the wording of *Elements* II.6 with a formulation referring to the area and the sides of a rectangle; it also changes the aim from the finding of unknown quantities (and, on the level of general aims, to train and display computational virtuosity) to the search for explanations or *causes*, in the idiom of Aristotelian philosophy (and, on the general level, search for and display of *wisdom*).

As to the time of the transfer, Euclid is not responsible; *Elements* II is certainly an older treatise, even though Euclid may have edited it to an

unknown degree. What little we know about the geometry of Hippocrates of Chios and Theodoros shows that the import will have taken place before their times (not necessarily the reformulation as critique); coins from Aegina, which in the fifth century had carried a “naive” geometric diagram, exhibit the diagram of II.4 (including a “critical” diagonal) from 404 B.C. onward, demonstrating that the topic was hot by then [Artmann 1988, 11]. The most likely epoch for the creation of the *Elements*-II theory is the mid- to late fifth century B.C.

Some of the traditional riddles are *not* reflected in *Elements* II even though they occur in Old Babylonian as well as Arabic sources: the rectangular problems $A+(l\mp w) = \alpha$, $l\pm w = \beta$; $A = \alpha$, $d = \beta$, and the circular problem $d+p+A = \alpha$. This has its good reasons: the three rectangular problems were always reduced to the basic types $A = \alpha$, $l\pm w = \beta$, and thus in need of no particular consolidation; the circular problem was excluded in the absence of a known ratio between d and p .

8. Intermediate positions

Less radical is the “translation” contained in Euclid’s *Data*. *Data* 84 does not *find* the sides of a rectangle from their difference and the area, it is true; but it shows that if the difference and the area are given, then even the sides themselves are given. In other words, it shifts the interest from solution to solvability; its relation to the old rectangle riddle is analogous to the relation between elementary algebraic theory and the practical solution of equations. The interest in solvability as a theoretical problem, however, is generalized and applied to questions far removed from what the Babylonians and later practitioners had imagined. This general investigation is hence no longer to be understood as “translation”, instead it is another reflection of the specific character of the Greek mathematical enterprise; it may be compared to that general investigation of the classes of irrational magnitudes and their mutual relations (*Elements* X) which grew out of the discovery of irrational ratios.

In Book I of Diophantos’s *Arithmetica*, a few propositions repeat the old riddles (evidently in numerical, not in geometrical formulation, and thus in the reverse translation of that conjectured by Neugebauer): $A =$

α , $l \pm w = \beta$ (Propositions 27 and 30); $A_1 \pm A_2 = \alpha$, $s_1 + s_2 = \beta$ (Propositions 28 and 29). Here, the immediate interest coincides with that of the practitioners: To find the numerical solution. In so far, we may speak of a genuine though free (and interpretive) translation. In contrast to what we find in Babylonian sources, however, theoretical reflection is also made explicit, not only in the sense that the problem is formulated both in general terms and in a numerical example used for the solution (general descriptions are attempted without much resulting clarity in some Old Babylonian texts on the interface between practitioners' and school culture, and with better outcome in a few late Babylonian tablets) but also by the formulation of diorisms telling the conditions for solvability; interestingly, these conditions are then told to be *πλασματικός*, which may (but need not) mean that they can be seen in a diagram, a *πλάσμα* (which indeed they can, namely in the diagram that follows from the Babylonian texts) – cf. the discussion in [Høyrup 1990a, 349f.].

9. *Dýnamis* – a loan translation?

Diophantos was not the founder of Greek algebra; for one thing, he tells that “it has been approved” (*ἔδοκιμάσθη*) to designate the second power of the unknown number as *dýnamis* (*δύναμις*), thus making it an “element of arithmetical theory” (*στοιχείον τῆς ἀριθμητικῆς θεωρίας*), i.e., *algebra* as treated by Diophantos [Diophantus 1893: I, 4]. This, and various other agreements, shows us that diverse passages in Plato's *Republic* refer to a second- and third-degree calculators' algebra, and that even this early algebra used the term *dýnamis* for the second power [cf. Høyrup 1990b, 368f.].

Other passages in Plato, Aristotle, the Hippocrates fragment, and the *Elements* (etc.) shows the *dýnamis* to be also a geometric term; its meaning has been much discussed, since some texts seem to understand it as a square and others as a square-root or the side of a square (and Plato's *Theaetetus* 147C7–148D7 mixes both senses). Complete analysis of the evidence [see Høyrup 1990b] shows the meaning to be a square parametrized by and hence tendentially identified with its side – a square *being* its side and *possessing* its area, whereas the *τετράγωνον*-square *is* its area

and *has* its side (in agreement with Euclid's definition of a "figure" as that which is contained by one or more boundaries).

In Babylonian mathematics, a similar concept is found: the *mithartum*, etymologically something like "a situation characterized by the confrontation of equals", is exactly this square "being" its side and having an area. This analogy does not necessarily entail that the term be borrowed – however strange the idea seems to us (and to later Greek geometers, who tended to eliminate the term), both mathematical cultures might have had a concept of the square referring primarily to the frame made up of equal sides. As pointed out by Szabó [1969, 46f.], however, both *dýnamis* and the verb *dýnasthai* (even this used in geometry) have connotations of equivalence and commercial value, together with the basic denotation of physical strength; exactly the same range of de- and connotations belongs with the verb *mahārum*, from which *mithartum* is derived. This still constitutes no proof that the *dynamis* be a calque of the Babylonian term (or rather, of some corresponding Aramaic term used by Near Eastern mathematical practitioners around 500 B.C.); but it strengthens the hypothesis, and goes well together with the evidence for a transfer of that very quasi-algebraic technique in which the Babylonian word had served.

10. Egypt and Phoenicia

Nothing in the above supports Herodotos's claim that Greek geometry was derived from the technique of Egyptian "rope stretchers". However, we should remember, firstly, that the area geometry found in *Elements* II (and X, and used in Apollonios's *Conics*, etc.) is not the whole of Greek geometry; secondly, that the Egyptian mathematical papyri inform us about Egyptian Middle Kingdom *area computation*, but not about mid-first millennium *mensurational techniques*, which is a very different matter. Our sources thus do not permit an informed opinion about Herodotos's claim.

Proclus's idea that "precise understanding of numbers" come from the Phoenicians may astonish, since the Greek notation for fractions is notoriously borrowed from Egypt, not from the Phoenician traders; but if we remember that Diophantos characterizes what he does as *arithmetic*, i.e., science of numbers, Proclus may be right, although in a sense that has not been suspected: the Greek calculators' algebra, indeed, is indubitably

derived from the lore of the Near East, and probably from contacts precisely with the Syrian coast land – i.e., Phoenicia.

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Slightly corrected 4 January 1998

Further minor correction 19 July 1999

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